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Quark number susceptibilities and the Wróblewski parameter

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Abstract. The Wróblewski parameter is a convenient indicator of strangeness production and can be employed to monitor a signal of quark–gluon plasma production : enhancement of strangeness production. It has been shown to be about a factor two higher in heavy ion collisions than in hadronic collisions. Using a method proposed by us earlier, we obtained lattice QCD results for the Wróblewski parameter from our simulations of QCD with two light quarks both below and above the chiral transition. Our first principles based and parameter free result compare well with the A–A data from SPS and RHIC.

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1 Introduction

As with many signals of quark–gluon plasma (QGP) production in relativistic heavy ion collisions, the basic idea behind enhancement of strangeness production [1] as a QGP signal is very simple. Recognising the fact that the strange quark mass is smaller than the expected transition temperature whereas the mass of the lowest strange hadron is significantly larger, it was argued that the production rate for strangeness in the QGP phase, $\sigma_{\rm QGP}(s\bar{s})$ is greater than that in the hadron gas phase, $\sigma_{\rm HG}(s\bar{s})$. While this energy threshold argument for strangeness production in the two phases is qualitatively appealing, one has to face quantitative questions of details for any meaningful comparison with the data. Applications of perturbative QCD need a large scale which could be either the temperature of QGP or the mass of the produced strange quark–antiquark pair. Since the temperature of the plasma produced in RHIC, or even LHC, may not be sufficiently high for perturbative QCD to be applicable and since the strange quark mass is also rather low, estimation of strangeness production by lowest order processes like $gg \to s\bar{s}$ could be misleading. Indeed, it is now well-known that even for the charm production, the next order correction to $gg \to c\bar{c}$ is as large as the leading order; such an order by order computational approach may be hopelessly futile for the much lighter strange quark.

A variety of aspects of the strangeness enhancement have been studied and many different variations have been proposed. One very useful way of looking for strangeness enhancement is the Wróblewski parameter [2]. Defined as the ratio of newly created strange quarks to light quarks,

$$\lambda_s = \frac{2\langle s\bar{s}\rangle}{\langle u\bar{u} + d\bar{d}\rangle} \qquad (1)$$

the Wróblewski parameter has been estimated for many processes using a hadron gas fireball model [3]. An interesting finding from these analyses is that λ_s is around 0.2 in most processes, including proton–proton scattering, but it is about a factor of two higher in heavy ion collisions. An obvious question one can ask is whether this rise by a factor of two can be attributed to the strangeness enhancement due to the quark–gluon plasma and if so, whether this can be quantitatively demonstrated by explicitly evaluating the Wróblewski parameter in both phases. Alternatively, one could just study how different the prediction actually is and learn about other physics effects from its comparison with data. We show below how quark number susceptibilities, obtained from simulations of lattice QCD, may be useful in answering such questions. Since these simulations correspond to equilibrium situations, one needs certain extra assumptions which we also discuss briefly.

2 λ_s from quark number susceptibilities

Quark number susceptibilities (QNS) can be calculated from first principles using the lattice formulation. Assuming three flavors, u, d, and s quarks, and denoting by μ_f the corresponding chemical potentials, the QCD partition function is

$$\mathcal{Z} = \int DU \quad \exp(-S_G) \prod_{f=u,d,s} {\rm Det}\, M(m_f, \mu_f) \quad . \qquad (2)$$

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Note that the quark mass and the corresponding chemical potential enter only through the Dirac matrix M for each flavor. We use staggered fermions and the usual fourth root trick [4] to define M for each flavor. Defining $\mu_0 = \mu_u + \mu_d + \mu_s$ and $\mu_3 = \mu_u - \mu_d$, the baryon and isospin densities and the corresponding susceptibilities can be obtained as

$$n_i = \frac{T}{V}\frac{\partial \ln \mathcal{Z}}{\partial \mu_i}, \qquad \chi_{ij} = \frac{T}{V}\frac{\partial^2 \ln \mathcal{Z}}{\partial \mu_i \partial \mu_j} \quad . \tag{3}$$

QNS in (3) are crucial for many quark–gluon plasma signatures which are based on fluctuations in globally conserved quantities such as baryon number or electric charge. Theoretically, they serve as an important independent check on the methods and/or models which aim to explain the large deviations of the lattice results for pressure $P(\mu{=}0)$ from the corresponding perturbative expansion. Here we will be concerned with the Wróblewski parameter which we [5] have argued can be estimated from the quark number susceptibilities:

$$\lambda_s = \frac{2\chi_s}{\chi_u + \chi_d} \ . \tag{4}$$

Note that the lattice simulations yield the real quark number susceptibility whereas for particle production its imaginary counterpart is needed. Indeed, λ_s above too needs the latter. However, one can relate the two and thus justify the use of lattice results in obtaining λ_s. Briefly, the argument [7] is as follows. Fluctuations in physical quantities, described by a perturbation in time, can be related to a generalized susceptibility for the corresponding operator for it. This is complex in general. Its imaginary part can be shown to determine the dissipation, i.e., the production of a strange quark–antiquark pair in our case. From the general properties of these susceptibilities, a Kramers–Kronig type relation between their real and imaginary parts can be obtained. Finally, making a relaxation time approximation ($\omega\tau \gg 1$), one finds that the ratio of the imaginary parts is the same as that of the real parts.

In order to use (4) to obtain an estimate for comparison with experiments, one needs to compute the corresponding quark number susceptibilities on the lattice first and then take the continuum limit. All susceptibilities can be written as traces of products of the quark propagator, $M^{-1}(m_q)$, and various derivatives of M with respect to μ. With $m_u = m_d$, the diagonal χ_{ii}'s can be written as

$$\chi_0 = \frac{T}{2V}\left[\langle \mathcal{O}_2(m_u) + \frac{1}{2}\mathcal{O}_{11}(m_u)\rangle\right], \tag{5}$$

$$\chi_3 = \frac{T}{2V} \quad \langle \mathcal{O}_2(m_u)\rangle, \tag{6}$$

$$\chi_s = \frac{T}{4V}\left[\langle \mathcal{O}_2(m_s) + \frac{1}{4}\mathcal{O}_{11}(m_s)\rangle\right]. \tag{7}$$

The operators $\mathcal{O}_2$ and $\mathcal{O}_{11}$ are defined by

$$\mathcal{O}_2 = \mathrm{Tr}\, M_u^{-1}M_u'' - \mathrm{Tr}\, M_u^{-1}M_u' M_u^{-1}M_u' \ , \tag{8}$$

$$\mathcal{O}_{11}(m_i) = (\mathrm{Tr}\, M_i^{-1}M_i')^2 \quad , \tag{9}$$

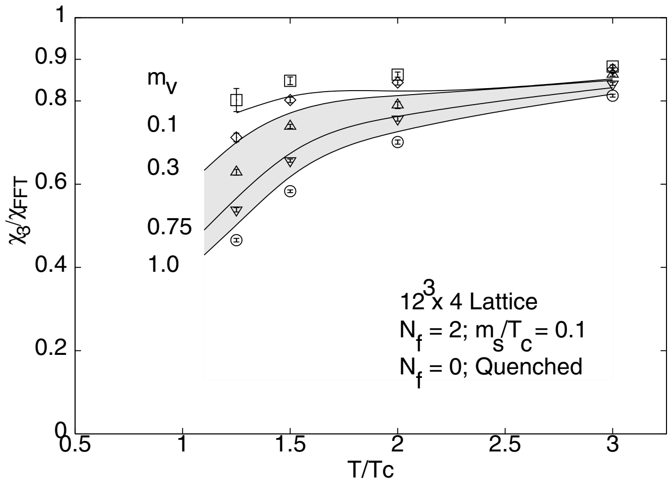

Fig. 1. Comparison of quenched (lines) and full QCD results (points) for the quark number susceptibility as a function of T/T_c. The valence quark mass (m_v) values in units of T_c are indicated. The grey band indicates the mass range for strange quarks

where $i = u$ or s. The traces are estimated by a stochastic method: $\mathrm{Tr}\, A = \sum_{i=1}^{N_v} R_i^\dagger A R_i / 2N_v$, and $(\mathrm{Tr}\, A)^2 = 2\sum_{i>j=1}^{L} (\mathrm{Tr}\, A)_i (\mathrm{Tr}\, A)_j / L(L-1)$, where R_i is a complex vector from a set of N_v, subdivided further in L independent sets. We use typically $N_v = 50$–100.

Figure 1 displays results [6] for the susceptibilities as a function of temperature in units of T_c, where T_c is the transition temperature. Normalized to the corresponding ideal gas results on the same lattice, i.e, the infinite temperature limit, results for QCD with two light dynamical quarks of mass 0.1 T_c are shown as points whereas the continuous curves correspond to the results in the quenched approximation. Note that the latter amounts to dropping the fermion determinant term in simulations which become orders of magnitude faster, and hence more precise, than the full QCD simulations. The valence quark mass m_v, appearing in (5)–(7), is shown in the figure in units of T_c. Note that T_c in these two cases differ by a factor of 1.6, but the results for the corresponding dimensionless susceptibilities as a function of the dimensionless ratio T/T_c differ by a few per cent only. Such a mild dependence on the number of dynamical flavors in the thermodynamic quantities has been a known feature in the temperature region away from the transition. Indeed, since the nature of the transition does depend strongly on the number of dynamical flavors, one expects significant differences near T_c. Encouraged by this behavior, we investigated the continuum limit for the quenched case by increasing the temporal lattice size from 4 to 14 in steps of two and extrapolating to infinite temporal lattices. The spatial lattices were also increased to maintain the aspect ratio constant. Figure 2 shows typical results of such continuum extrapolation at $T = 2\ T_c$. The continuum results for the light quark susceptibility thus obtained in the quenched approximation are exhibited in Fig. 3 for small m_v. The bands marked by "HTL" and "NL" show the analytic results of [8] obtained in successively better approximations respectively.

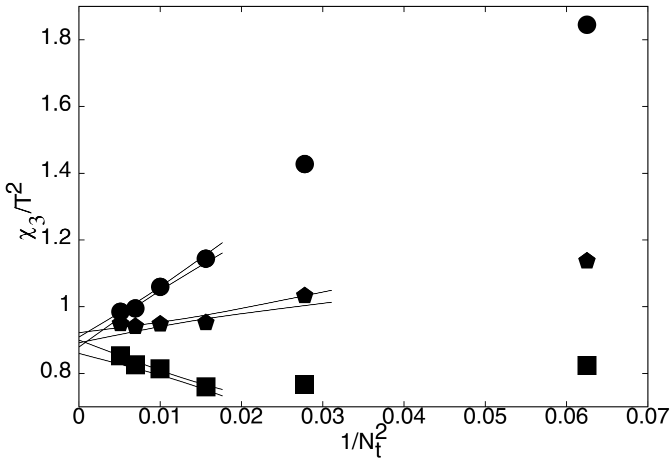

Fig. 2. Continuum extrapolation results at $2T_c$. The circles and pentagons are for two different actions and the squares are for another method. All must lead to the same result in the $N_t \to \infty$ continuum limit, i.e., for vanishing lattice spacing

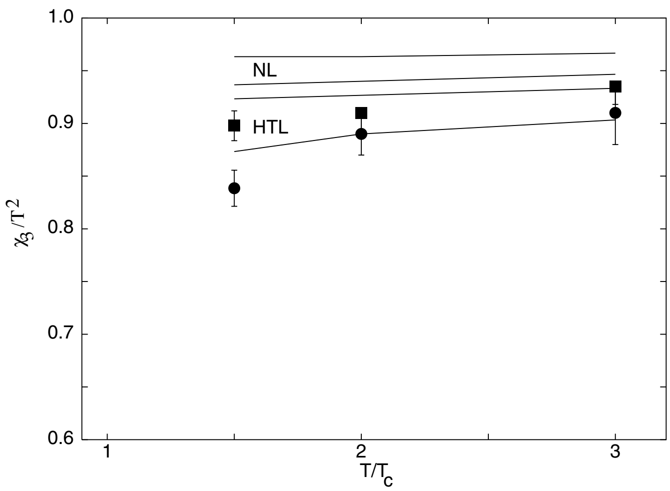

Fig. 3. Quenched QCD results for χ_3 in the continuum limit. The points denote different input lattice actions. The curves are analytic results in different approximations and are from [8]

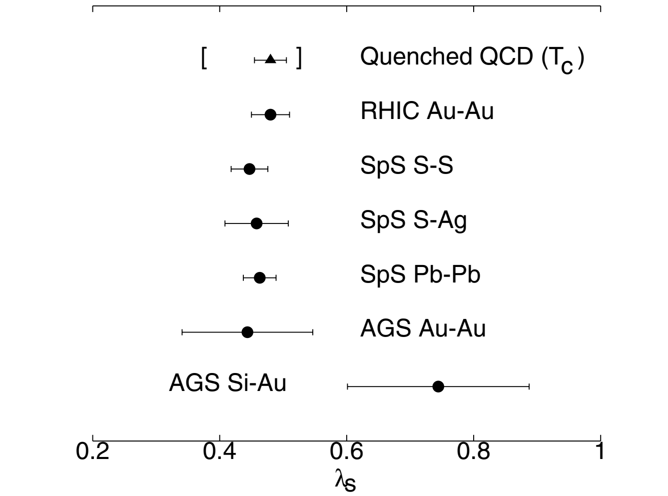

Fig. 4. Comparison of the λ_s from our quenched QCD in the continuum limit with RHIC and SPS experiments estimates [3]

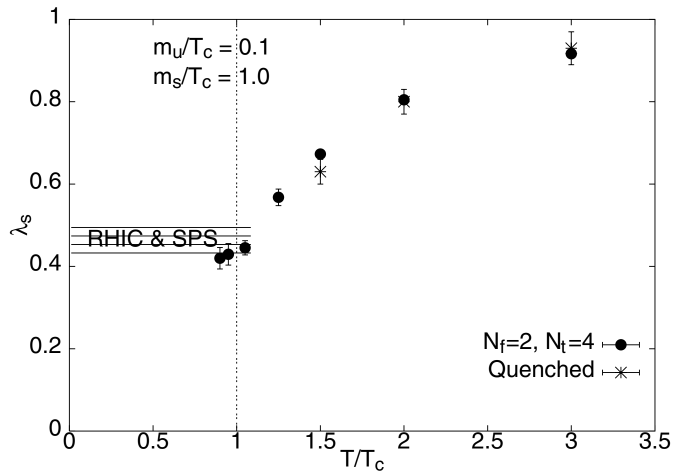

Fig. 5. λ_s as a function of temperature for full (filled circles) and quenched QCD (crosses). The experimental results from SPS and RHIC are indicated by the marked band

The strange quark susceptibility in the continuum limit can be obtained from the same simulations by choosing the valence quark mass m_{v} appropriately. We use $m_{\mathrm{v}}/T_{\mathrm{c}} = 1$, corresponding to the canonical choice for the strange quark mass and the T_{c} in the full theory. In view of Fig. 1, which shows that the results for full and quenched QCD differ by only a few per cent when compared in the appropriate dimensionless variable, we expect that this choice of $m_{\mathrm{v}}/T_{\mathrm{c}}$ for strange quark will lead us to results of relevance to the world of full QCD. Using (4), $\lambda_s(T)$ can then be easily obtained. These were extrapolated to T_{c} by employing simple ansätze. The resultant $\lambda_s(T_{\mathrm{c}})$ in quenched QCD is shown in Fig. 4 along with the results obtained from the analysis of the RHIC and SPS data in the fireball model [3]. The systematic error coming from extrapolation is shown by the brackets. The agreement of the lattice results with those from RHIC and SPS is indeed very impressive.

The nice agreement needs to be treated cautiously, however, in view of the various approximations made. Let us list them in order of severity.

(1) The result is based on quenched QCD simulations and extrapolation to T_{c}. As seen from Fig. 1, the quark number susceptibilities, and hence $\lambda_s(T)$, are expected to change by only a few per cent. Since the nature of the phase transition does depend strongly on the number of dynamical quarks, a direct computation near T_{c} for full QCD is desirable. We are currently making such a computation and have some preliminary results for full QCD with two light dynamical quarks for lattices with four sites in temporal direction. These are shown in Fig. 5 along with the continuum quenched results for $\lambda_s(T)$ and the band for experimental results. While the emerging trend is encouraging, further exploration with varying strange quark mass, temporal lattice size (to obtain continuum results) and spatial volume is still necessary.

(2) The experiments at RHIC and SPS have non-zero albeit small μ whereas the above result used $\mu = 0$. Based on both lattice QCD and fireball model considerations, λ_s is expected to change very slowly for small μ. This can,

and should, be checked by direct simulations.
(3) As argued above, particle production needs the imaginary counterpart of what one obtains from simulations. The relation between the ratios of real and imaginary parts was obtained under the assumption that the characteristic time scale of quark–gluon plasma are far from the energy scales of strange or light quark production. Observation of spikes in photon production may falsify this assumption.

3 Summary

Quark number susceptibilities contribute in many different ways to the physics of the signals of quark–gluon plasma in heavy ion collisions at SPS and RHIC. They can be obtained from first principles using lattice QCD. This offers a quantitative control and check of these signals and thus QGP itself. In particular, the continuum limit of χ_u and χ_s, which we obtained in quenched QCD, leads to a temperature dependent Wróblewski parameter, $\lambda_s(T)$. Its extrapolation to T_{c} appears to be in good agreement with results from SPS and RHIC. First full QCD results near T_{c} confirm this as well, although many technical issues, e.g, finite lattice cut-off or strange quark mass, need to be sorted out still.

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